Technical Comments

Comment on "Optimal Space Flight with Multiple Propulsion Systems"

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In a recent article, Hazelrigg[1] presented a thrust switching criterion that may be used to select the optimum thrust and I_{sp} levels (out of several available combinations, including coast) at all points along a space flight trajectory. A conventional "switching function" K_i is computed for each of the n possible thrust combinations; the value of $i(1 \leq i \leq n)$ is then chosen to maximize the product $K_i\beta_{i\ \max}$ or, equivalently, the Hamiltonian function H.

This is an interesting and useful approach. It must be pointed out, however, that essentially similar contributions have appeared previously in the 1967–1968 literature. The object of this Comment is to bring two prior and relatively thorough treatments to the attention of readers who may be contemplating further development of the subject. Papers by Dickerson and Smith,[2] and Fishbach,[3] for example, resulted in switching criteria identical or mathematically equivalent to the $K_i\beta_i$ function of Ref. 1 [i.e., compare Fig. 1 (Fig. 1 of Ref. 1) with Fig. 2 (Fig. 4a of Ref. 3)]. Moreover, Ref. 2 analytically developed the optimal thrust sequencing policy; the same result was illustrated numerically (for several values of n) in Ref. 3. References 1 and 2 both showed a solar-electric power source in their illustrative examples, while Ref. 3 showed the constant power case. Reference 3 further compared the resultant acceleration histories, J values, and propellant requirements with those corresponding to the variable-thrust case, and showed that variable thrust performance can be rather closely approached using as few as three different thrust levels.

The main features of these contributions are summarized in Table 1. We suggest that the points covered are well known by now and that future contributions could more appropriately be directed toward either significant theoretical extensions or realistic engineering applications.

Table 1 Some recent results on multiple-thrust-level trajectories

	Paper		
Result	Hazelrigg,[1] 10/68	Dickerson and Smith,[2] 8/68	Fishbach,[3] 8/67
Development of βK criterion (or equivalent)	Yes	Yes	Yes
Thrust sequencing policy	Analytical: illustrative analytic example for the field-free case, $n = 2$	Analytical proof	Numerical demonstration for n = 2, 3, 5, 6
Power source illustrated	Solar-electric	Solar-electric	Constant power
Efficiency illustrated	Constant	Variable	Not considered
Comparison with variable thrust case	No	No	Yes; $a(t)$, $a^2(t)$, $J(t)$, and M_p as functions of n

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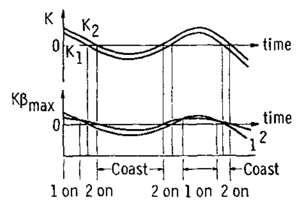

Fig. 1 Switching function histories taken from Ref. 1.

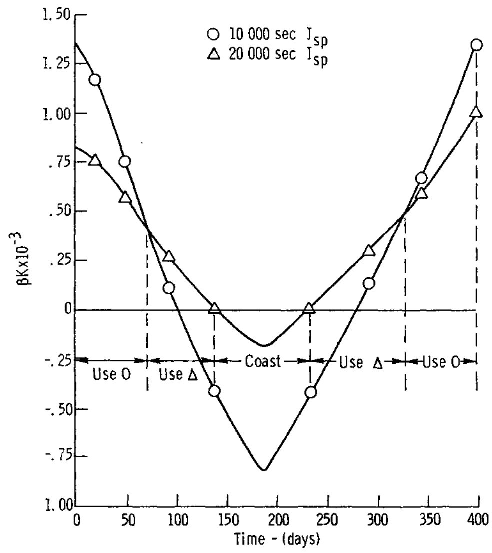

Fig. 2 Switching function histories taken from Ref. 3.

References

[1] Hazelrigg, G. A., Jr., "Optimal Space Flight with Multiple Propulsion Systems," *Journal of Spacecraft and Rockets*, Vol. 5, No. 10, Oct. 1968, pp. 1233–1235.

[2] Dickerson, W. D. and Smith, D. B., "Trajectory Optimization for Solar-Electric Powered Vehicles," *Journal of Spacecraft and Rockets*, Vol. 5, No. 8, Aug. 1968, pp. 889–895.

[3] Fishbach, L. H., "Multiple Thrust Level Trajectories for Minimum Propellant Consumption," M.S. thesis, 1967, Case Institute of Technology, Cleveland, Ohio; also TM X 52322, Aug. 1967, NASA.

Erratum: "Natural Frequency of Longitudinal Modes of Liquid Propellant Space Launch Vehicles"

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In the foregoing paper expressions were derived for use in calculating longitudinal modes of thin shell tanks that are built up from conical frustums and which contain liquid.

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